Didactic Strategy for Studying the Mathematical Concept of Parabola based On the Law of Light Reflection

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Abstract

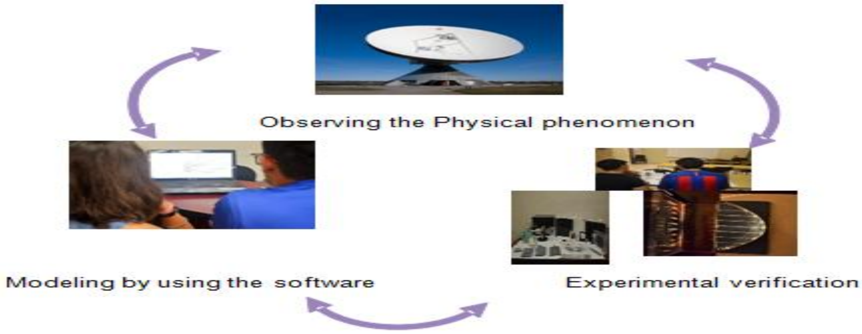
The research was aimed at devising a didactic strategy oriented to the comprehension of the mathematics concepts based on the law of reflection, particularly within the area of Analytic Geometry. For that purpose, the proposal was to study the physical phenomenon of the parabola (parabolic antennas), its mathematic modeling by means of CabriOlus II software, and its experimental testing of verification. A prototype was designed and built for verifying the focal property of the parabola, location of the focus and its relation to the straight side. The suggested procedures (observation, modeling, and experimental testing) relate the learning of mathematics concepts to physical phenomena of real life.

Keywords: Mathematical concept, Analytical Geometry, parabola, mathematical modeling, ICT, experiment.

Introduction

A teaching strategy was devised to face students' learning difficulties in understanding the concepts of the section of the conics within the Analytical Geometry course, taught at the Faculty of Physical-Mathematical Sciences. This strategy results innovative for its focus on an experimental approach, and the use of ICT as a tool for learning. The adequate use of methods, procedures and material aids supported by ICTs (Information, Communication and Technology) and the use of ICTs as a learning tool promote the students' interest to research, find out, verifying the findings, communicating the new knowledge and arguing their fundamentals with a critical and scientific spirit, as well as developing the related capacities (Bricall, 2000). Experimentation, Osorio (2004) has suggested, allows the student to contrast hypotheses related to physical phenomena studied in theory, proved them and construct their own knowledge in actual practice. Likewise, the potentials of considering mathematical science and didactics in a dialectical unity were taken into account for the understanding of the mathematical concepts of conic sections. The framework of the research herein describes includes both notions dealing mathematic concepts and its corresponding didactics (Mola, 2014; Pecharromán, 2014 & 2013; Godino, Contreras, & Font, 2006; Rodríguez, 2003; Alcalá, 2002; and Bricall, 2000). Ways to study the mathematical concept of the parabola in an integral form are fully described. These ways are: observing real-life objects, ICT supported modeling of the phenomena by using Cabri Plus 11 software and verifying findings experimentally (Figure 1).

Figure 1: Interrelated ways of learning mathematics concepts.

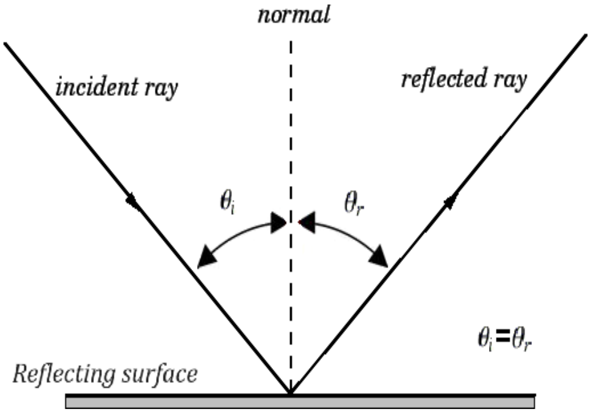


The first alternatives consist of making an external observation of actual objects or phenomena for instance, a Parabolic Antenna, moving to a description of the perception of the antenna through modeling with the use of (software Cabri Plus 11), and finally, verify the accurateness of the object represented by conducting experiments. In this particular case, an optical prototype device for studying conic sections corroborates what was previously studied with the software. A second alternative might be leading the students to face problematic situations, which can give way to verification through experimentation, and then proceed to the modeling of such situation. Finally, a third alternative might be approaching physical phenomena through experiments where students elaborate questions and problematic situations that make possible the understanding through its modeling with the use of the software. As mentioned above. A Cabri Plus 11 software was used and an optical prototype was designed to check the focal distances of the conic sections of the parabolas. The following is a description of the procedures to be followed to put the proposed strategy into practice.

Figure 2: Parabolic Antenna.

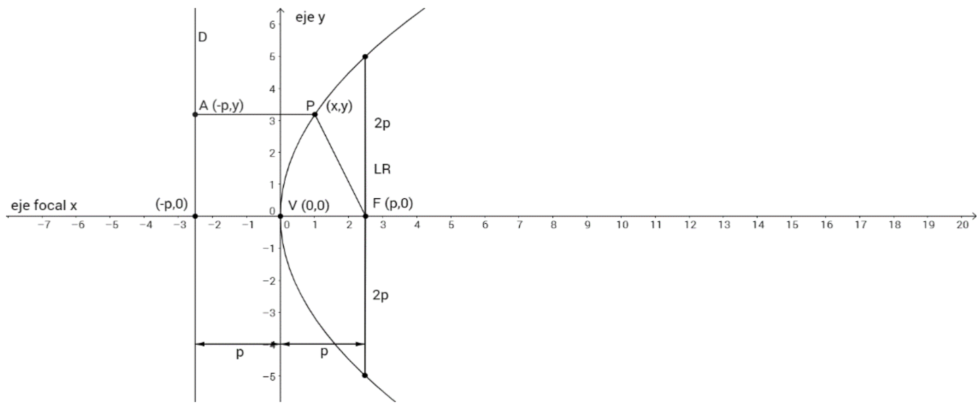
Figure 2 shows a parabolic antenna a practical illustration of the concept of the parabola. The students are asked to examine the figure, even the real object if possible, and are questioned about its operation, mathematical modeling and its application. This procedure is intended to explore students' preconceptions. One of the preconceptions presented by most students is the Law of Reflection of how the microwaves are reflected and crossed by a point called the receiver or focus of the parabolic antenna. Therefore, the two principles of the law of reflection (Figure 3) are stated below.

Figure 3: Law of Reflection.



The first principle states that the incident ray, the normal ray, and the reflected ray must be contained in the same plane.The second principle establishes that the measurement of the angle of incidence is equal to the angle of reflection, as shown in figure 3. That is to say that θi = θr. Where the subscripts i and r correspond to the incident and reflected respectively. From here on, the behavior of light rays and their resemblance to microwaves on a reflecting surface section of the parabola is explained. Both light rays and microwave waves reflecting on a surface are reflected according to the law of reflection. In parabolic antenna modeling students are instructed how the Cabri Plus 11 software works and how to operate it, once it is installed. Then the parabola is modeled and tangent lines to the surface and perpendicular to it are drawn. Only those straight-lines located at the straight side (RL) of figure 5are shown.

Figure 4: The Parabola of vertex at the origin and focal axis X.



The following is a description of the corresponding equations. Likewise, the equations used for the tangent straight-lines of the parabola and the perpendicular to it are given. According to the Analytic Geometry written by Lehmann, the parabola is defined as the "geometric place of a point moving on a plane so that the distance from straight-line located on the plane is always the distance of a fixed point on the plane outside that line. The fixed point is called 'F' and the direct straight line of the parabola is termed 'D' ". The description of the parabola with a vertex at the origin and the axis coinciding with the x-axis or the y-axis is illustrated in figure 4. The focal axis of the parabola is the straight line crossing "F" and being perpendicular to "D". The point "V" (0,0), being the medial point of the segment "DF", represents the point where the parabola crosses the focal axis. The coordinates of the Focus "F" (P, 0) is a point located on the focal axis at a distance from the focal vertex equal to the vertex of the direct. The direct "D" (-p, 0) is a perpendicular straight-line to the focal axis of the parabola located at the same distance from the vertex than the vertex to the focus. The straight line segment joining the two points of the parabola is called cord, particularly the focal cord perpendicular to the axis is called right side. Given any point "P" (x, y) at the parabola, let's draw a segment "A" perpendicular to "D", as shown in the figure, then, by definition, the point "P" must satisfy the geometric condition 1FP1=1PA1. This equivalence yields the equation y^2=4Px.The parabola equation with vertex at origin and axis X as the parabola axis.

P = Focal distance magnitude
4P= Magnitude of the right side RL.
The following are the equations used to obtain the straight-line tangent and perpendicular to the Cabri Plus 11software modeling of the parabola (figure 5).
The straight line and slope equations are the following:

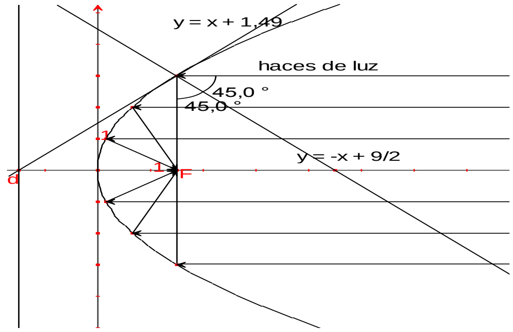
y-y_1=m(x-x_1) . Equation of the slope point straight-line.

m=(y_2-y_1)/(x_2-x_1). Equation of the slope of the straight-line.

m_1 m_2=-1m_1 ym_2arepenpendicular slopes.

Light beams that strike parallel to the axis of the parabola are reflected according to the law of reflection and cross at a point called the focus of the parabola. However, there is a parallel beam that relates to the equation of the parabola, which strikes a point at the surface, and that coincides with the parallel straight-line drawn in figure 4. It is reflected with a similar striking angle and crosses the axis of the parabola and the focus up to a point of its surface and reflected parallel to the axis as it will be proved soon at the experimental activity. In the same way, it is possible to explain that such light beams striking parallel to the axis of the parabola are reflected accordingly to the law of reflection, crossing at a point called parabola focus. In correspondence with the equation of the parabola y^2=4Px, where 4p is defined as the right side (RL) magnitude (figure 5), it can be observed that such magnitude is related to the parallel beam striking at a point of the surface with respect to the normal straight-line at 45 degrees and reflected with the same angle crossing perpendicularly the axis of the parabola, crosses the focus, reaches a point of its surface and is reflected parallel to the its axis (this imaginary straight-line coincides the straight line drawn perpendicular at figure 5). The experimental activity below will illustrate this.

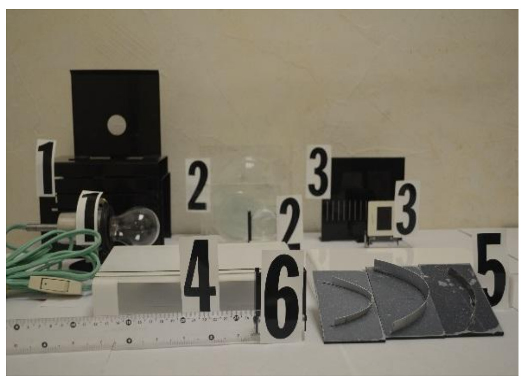
5: The Parabola modeling using Cabri Plus 11.



Experimental activity. An optical prototype was designed and constructed for the conic sections of the parabola, (Figure 6), and its components are described as follows:

1. A light source with its respective cover.
2. Collimating lenses.
3. Grids of 3 and 12 divisions.
4. Beam collecting table.
5. Parabola sections of 2, 3,4 centimeters of the straight side (RL).
6. Rule scaled in cm and inches.

Figure 6: An optical prototype of parabolic conic sections.



According to the users' manual of the experimental prototype of parabolic conic sections, the components of the equipment should be placed as illustrated in figure 7.

Figure 7: The Experimental arrangement for obtaining light beams.

One of the three different images shown in Figures 8, 9 and 10 is obtained.

Figure 8: Convergent light beam.

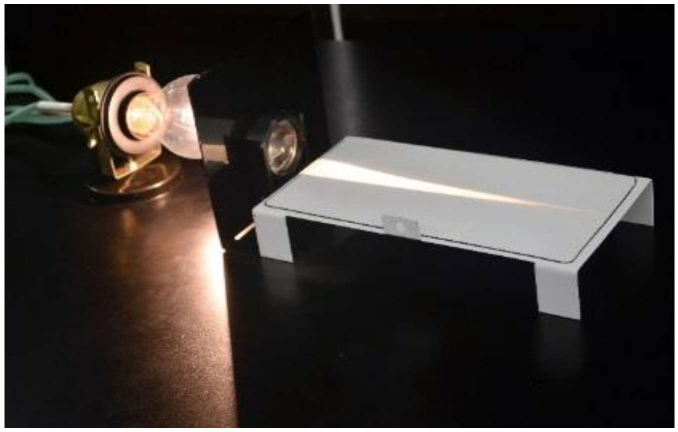


Figure 9: Divergent light beam.

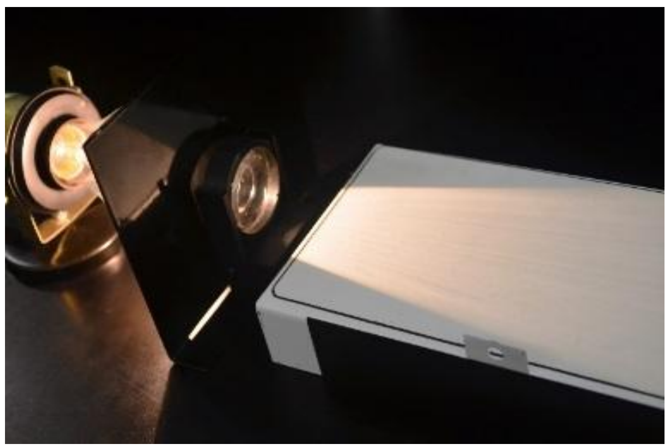
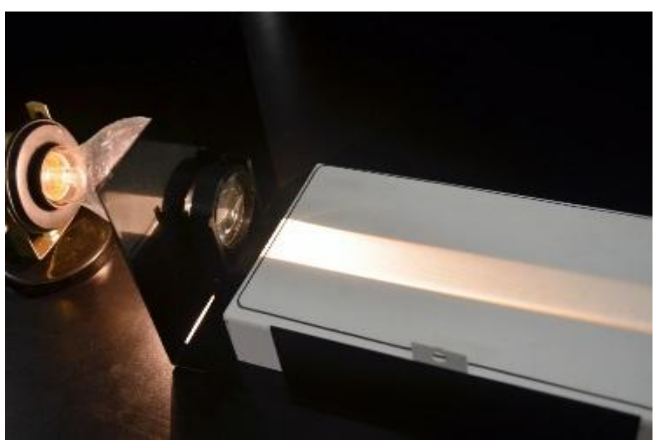


Figure 10: Parallel light beam.



These depend on the position of the light source relative to the collimating lens. If the resulting image is convergent beams, the light source approaches the collimating lens until a uniform beam of light is obtained; in the same way, if the beam of light is divergent, the light source of the collimating lens is withdrawn until a parallel beam of light is obtained. Later, a three-line grid is interposed to obtain three parallel beams on the light collecting table (Figure 11).

Figure 11: Using a three-line grid produces three parallel beams.

It is also possible to obtain a larger number of parallel beams by using grids of a larger number of lines either horizontal or vertical rays (Figure 12).

Figure 12: Using a ten-line grid produces ten horizontal parallel beams (top) or ten vertical parallel beams (down).

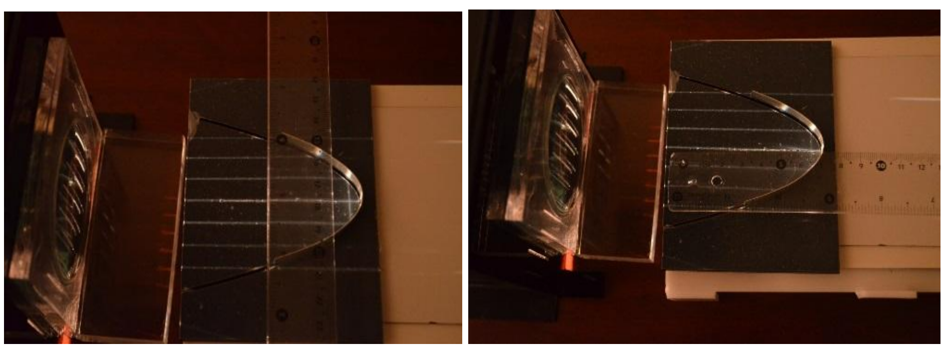
Figure 13: The 2, 4, and 6 centimeters straight side section of the parabola.

The arrangement of Figure 12 will be used to carry out the experimental part. Figure 13 shows the section of straight side parabolic (RL) 2, 4 and 6 cm. These sections of the parabolas are interposed on the beam collecting table and the intersection of the reflected rays in the focus and the straight side of the parabola is observed (figure 14), as demonstrated at the theoretical activity with the software Cabri Plus 11.

Figure 13: The 2, 4, and 6 centimeters straight side section of the parabola.

Figure 14: Images of the focus and the straight side section of the parabola.

Figure 15: Images of straight side and focal distance of the parabola measurement.



To determine the magnitude of the straight side and its focal length, the scaled ruler is placed as shown in Figures 15, 16 and 17, and the measurement of each one is read. Naturally, such measurement yield approximately values as compared to the accurate values of the parabola equation.

Figure 16: Images of straight side and focal distance of the parabola measurement.

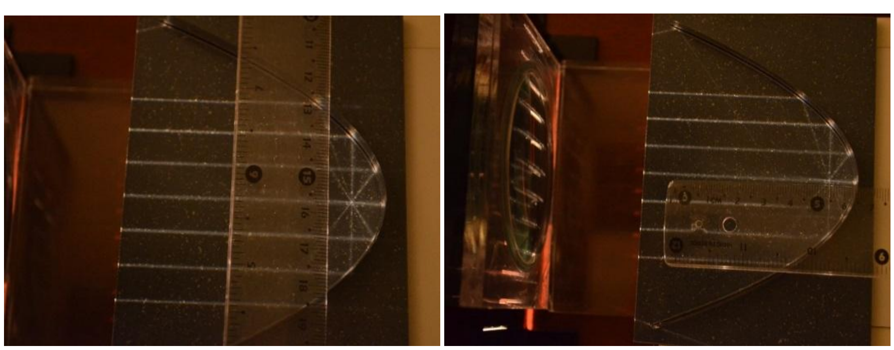
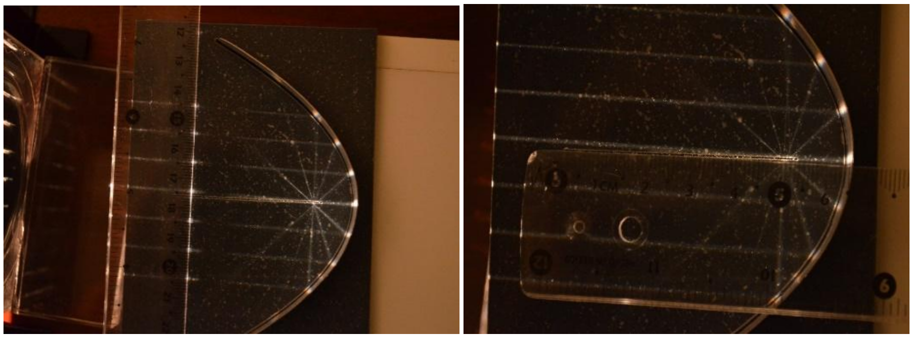
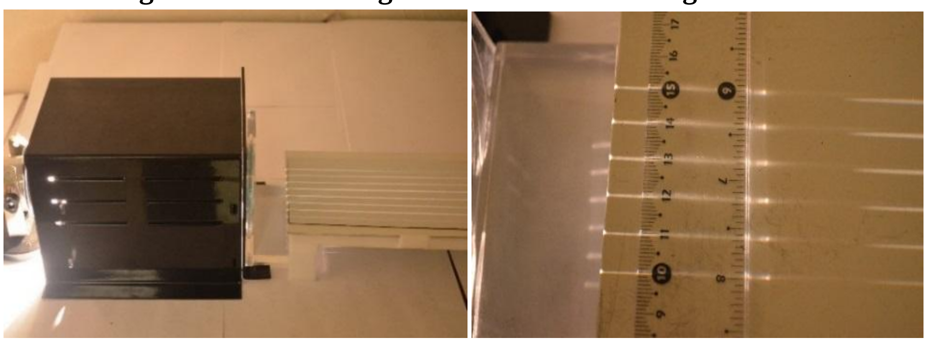


Figure 17: Images of straight side and focal distance of the parabola measurement.



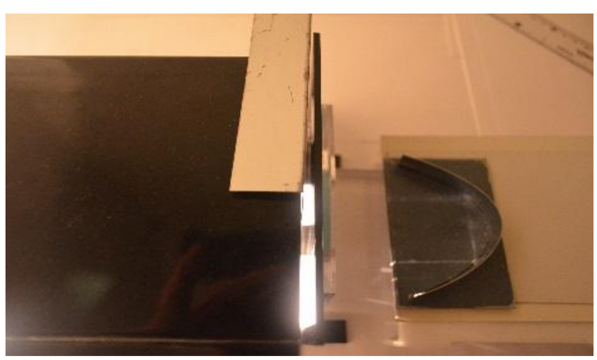
RL. 6 centimeters Focal distance 1.5 centimeters

Figure 18: Measuring the distance between light beams



On the other hand, another way of measuring the straight side of the parabola is proposed. At the arrangement illustrated in figure 12, place a scaled ruler and measure the spacing of the beams (figure 18); then remove the scaled ruler and insert one of the sections of the parabola on the beam collecting table instead, then observe how many beams there are from end to end of the straight side. The measurement depends on the number of beams and their separation from each beam (figure 12). For example, in Figure 18, the spacing of each beam is approximately one centimeter and accordingly, from Figure 16 it is observed that there are four beams on the straight side; therefore, their measurement is 4 cm. The modeling of the straight side of the parabola was made by using the cabri plus 11 software. In addition, the experimental equipment (figure 12) serves to prove what was previously modeled. For that purpose, the light beams are interrupted leaving one beam to strike into a point at the straight side of the parabola (figure 19). It can be observed that this beam is reflected perpendicularly to the axis of the parabola, reaching the other side of the straight side and being reflected parallel to the incident beam (figure 19).

Figure19: Image of a parallel beam striking at the straight side section of the parabola.



Summing up, the proposed didactic strategy for studying the parabola comprises three stages, first observing of real objects and phenomena; second, carrying out the corresponding modeling with the software Cabri Plus 11, and finally verifying the findings by means of experimental equipment, here the first route has been previously presented.

This paper has described the process in the order mentioned afore. However, it is also possible to reverse the order, that is, to complete the experimental activity first and then move to modeling with the software Cabri Plus

Conclusions

The proposed strategy describes how to approach the mathematical concept of the parabola conic. The illustrated procedures open new possibilities for studying other conic sections such as hyperbolas and ellipses under the same scheme. The presentation of new Analytical Geometry contents in a didactic way might prove to be useful for the students and eventually arises motivation to explore several fields of Mathematics. The suggested set of procedures (observation, modeling, and experimental testing) relate learning to real life situation, thus students are expected to figure out the mathematic concept of parabola out of real practice, instead of assimilating predesigned notion usually learn by heart.

Sponsoring information: Facultad de Ciencias Físico Matemáticas, Universidad Autónoma de Nuevo León.

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