Impact of a Field-Based Mathematics Methods Course on Preservice Elementary Teachers' Specialized Content Knowledge: The Case of Area and Perimeter

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Abstract

This case study examined the impact of a field-based mathematics methods on the specialized content knowledge (SCK) for teaching area and perimeter with specific focus on representational competence and questioning strategies. Three preservice elementary teachers (PSETs) were purposefully selected from the 15 students who were enrolled in the course. They first solved the Rectangular Garden Task outside of class and presented their solutions for a whole class discussion. They then planned four lessons based on the task and taught it to students in their field placements. The data sources consisted of their solutions to task, lesson plans, video recordings of their teaching, field notes and their written reflections. The results revealed that they all showed growth in their representational competence and questioning strategies as a result of engaging in solving the task and teaching the four lessons. However, they experienced challenges in asking open-ended and follow-up questions and wait time.

Keywords: specialized content knowledge, area and perimeter, practice-based methods course, representational competence, questioning strategies.

1. Introduction

The goal of mathematics teacher preparation programs is to equip prospective teachers for the broad spectrum of mathematical learners they will encounter. Part of that process includes developing their Specialized Content Knowledge (SCK), defined by Ball, Hill, and Bass (2005) as a deeper understanding of mathematics that allows teachers to explain new ideas, work problems in multiple ways, and analyze student solutions. SCK is further defined as "mathematics that is useful in teaching but is not typically taught in conventional mathematics classes either at the high school or postsecondary levels" (McCrory, Floden, Ferrini-Mundy, Reckase, &Senk, 2012, p. 598). McCrory et. al. (2012) go on to say that for most teachers, SCK is not included in their formal mathematics education courses. Some examples of SCK include "looking for patterns in student errors, understanding different interpretations of the operations, appreciating the difference between models, how to choose, make, and use mathematical representations effectively, and how to explain and justify one's mathematical ideas. (Ball, Thames & Phelps, 2008, p. 400).

Although SCK is critical to effective mathematics teaching and learning, it is often challenging to develop since it focuses on employing variety of instructional strategies that allow students gain access to the mathematics concept being taught (Morris& Hiebert, 2009). According to Ding and Capraro (2013), the acquisition of SCK demands possession of well-connected mathematical knowledge and how children learn which is lacking among PSETs. As such, it is important to examine ways to support the development PSETs' SCK to enable them better teach their future students. The purpose of this article is to examine the impact of a field-based mathematics methods course on the development of PSETs' SCK for teaching area and perimeter in relation to representational competence and questioning strategies. By documenting PSETs' knowledge growth successes and challenges in learning to teach area and perimeter, this study aims to contribute a small but significant piece of knowledge base of mathematics teacher education (Morris& Hiebert, 2009) particularly in the teaching of area and perimeter.

2. Literature Review

2.1. Teaching Area and Perimeter

Practical concepts, such as area and perimeter, have an important part in today's school mathematics curricula and are widely used in everyday life. Research indicates that students and preservice elementary teachers (PSETs) struggle with and harbor misconceptions regarding these topics.

For example, Livy, Muir and Maher (2012) found that preservice teachers were confused when they were asked whether the area of a rectangle increases with increase in the perimeter. Also, a considerable number of the PSETs agreed that as the area of the rectangle increased so was the perimeter. Yeo (2008) found that some of the teachers in their study confused area and perimeter and assumed a constant relationship between the two measures. Furthermore, some PSETs perceived area and perimeter as purely an application of formula without understanding of what area and perimeter actually mean. Moyer and Husman (2006) pointed out that students' misunderstanding of perimeter and area may be due to the emphasis placed on the application of formulas and procedural techniques.

As a result of the misconceptions and confusion held by students and PSETs regarding area and perimeter, researchers such as Martin and Strutchens (2000) have suggested the use of physical and visual representations to help make sense of the concepts. They also suggest engaging students in investigating contextual tasks that involve finding area or perimeter with either the area or perimeter fixed. Tasks of this nature provides opportunity for conceptual exploration as opposed to memorization and application of formulas.

In light of the misconception PSETs have about area and perimeter and mindful of the challenges inherent in teaching these concepts, it seemed timely to focus this study on the development of PSETs SCK for teaching area and perimeter through contextual task. The *Fixed Perimeter Garden Task* which was adapted from Sullivan and Lilburn (2002) and Van De Walle, et al. (2019) was used for this study. The study of perimeter-area relationships not only encourages important mathematical explorations, but it also lends itself well to authentic applications beyond the classroom. In addition, the New York State Next Generation Mathematics Learning Standards (NGMLS, 2019) and the Common Core State Standards (CCSS, 2010) clearly expects third grade students to "identify rectangles with the same perimeter and different areas or with the same area and different perimeters" (NGMLS, p. 53). Therefore, if PSETs are to teach in a manner they will be teaching, then it is imperative and important for them to develop SCK that will enable them to unpack mathematical concepts in meaningful ways.

2.2. Representational Competence

Representational competence in mathematics is the ability to use representations meaningfully to understand and communicate mathematical ideas and to solve problems (Huinker, 2015). Ball and colleagues (2008) posit that teachers who have SCK have the knowledge and ability to select appropriate forms of representation for a particular purpose and linking it to the underlying ideas and other representations including both symbolic and non-symbolic. Fennell and Rowan (2001) pointed out that when students are able to represent a mathematical task in a way that is meaningful and make sense to them, the task becomes more accessible. This helps them better organize their thinking and facilitate the use of various approaches that may lead to a clearer understanding and a solution.

One of the ways that teachers can help students make sense of mathematical situations is by providing contextualized problem situations that are rich in various representations of mathematical concepts. By using different representations of mathematical concepts, we give students many opportunities to develop intuitive, computational and conceptual knowledge (Moyer, 2001). According to Tripathi (2008) using "different representations is like examining the concept through a variety of lenses, with each lens providing a different perspective that makes the picture [concept] richer and deeper" (p. 439). The National Council of Teachers of Mathematics (2014) have provided five modes of representations and the critical connections among the representations that are needed for developing students understanding of mathematics. The representations are physical, visual, contextual, verbal and symbolic. In considering implications of representational competence for classroom practice, Huinker (2015) suggest the following student abilities as illustrating such proficiencies: (1) being able to convey a mathematical idea in various forms; (2) knowing when and why it is appropriate to use particular mathematical representations; (3) being able to translate between and within modes of representations; (3) being able to us representations flexibly to solve problems

2.3. Questioning Strategies

Good questioning skills are part of the artistry of teaching. Well-crafted questions can assist students in digging deeper for more thoughtful responses. They can allow students to reflect on their own thought processes and to develop the ability to clearly articulate their thinking. Furthermore, skillful questioning leads students to make their own discoveries and create their own learning (Gallagher, 2015). According to NCTM (2014), "effective mathematics relies on questions that encourage students to explain and reflect on their thinking as an essential component of meaningful discourse" (p. 35). Research shows that the kinds of knowledge children construct and communicate during a mathematics lesson may be prompted by teachers' questions. Teachers who can question effectively are better able to discern the range and depth of children's thinking. A good question may mean the difference between constraining thinking and encouraging new ideas, and between recalling trivial facts and constructing meaning (Fuson & Sherin, 2004; Moyer & Milewicz, 2002).

Ball, Sleep, Boerst & Bass (2009) suggest criteria for mathematics teacher education context to be used when determining high-leverage practices for preservice teachers. Can the practice be taught to and practiced by beginning teachers in field-based settings? Is the practice accessible to learners of teaching and be able to be revisited many times with increased sophistications? Evidence from research suggests that the practice of teacher questioning does need to be taught rather than assuming teaching experience alone will build the competence (Schwartz, 2015). Also, Ball et al., (2008) posit that teachers who have SCK have the knowledge of or ability to pose questions that support students in building mathematical ideas. It is therefore important to develop PSETs' questioning strategies as part of a mathematics methods course (Chin, 2004).

3. Methodology

This study, which is part of a field-based mathematics methods course, utilizes qualitative case study research design (Stake, 1995). The researcher, who was also the course instructor, decided to conduct this study in order to gain an "in-depth understanding of the situation and the meaning for those involved" (p. 19). The situation in this context is the development of PSETs' SCK for teaching area and perimeter in the domain of representational competence and questioning strategies. The research question that guided this study was: *Which learning venues in a field-based mathematics methods course supported the development of PSETs' SCK for teaching area and perimeter in the domain of representational competence and questioning strategies?*

3.1. Participants

This study took place during the fall of the academic year in a suburban K-4 school that serves a predominantly Hispanic and African American student population. Of the fifteen PSETs who were enrolled in the course, three PSETs (Jessica, Sandra and Maya, all pseudonyms), were purposefully selected since they were the only students who were assigned to a third-grade classroom in a suburban elementary school that serves a predominantly Hispanic and African American community. Jessica and Sandra came from Caucasian background while Maya was of Hispanic origin.

3.2. Course Design

This four-credit-hour field-based mathematics methods course for Grade 1-6 PSETs met once a week for three hours during a 15-week semester at an elementary school in a suburban community that serve predominantly Hispanic and African American students. During the first hour PSETs worked with their respective host teachers in a number of capacities including observing the host teacher teach, working one-on-one with students who needed remediation and plan and teach lessons to a small group of students or a whole class. After the classroom experience, they spend the rest of the time with the course instructor engaging in one or more of the following activities: discussing learning theories, reflecting on their classroom experiences, addressing curriculum issues, examining effective teaching practices, solving problems and presenting their solutions to mathematical tasks from previous week.

As part of the process of developing PSETs' SCK for teaching, they were assigned non-routine tasks to solve outside of class over the course of the semester. Their solution strategies were discussed in class and each PSET was required to plan and teach four lessons based on the task appropriate for their assigned grade level. The purpose of solving the task first was to get to know the mathematical ideas they hold, the skills they possess, and the contexts in which these mathematical ideas are understood" (Conference Board of the Mathematical Sciences, 2001, p. 17). Hedges, Huinker and Steinmeyer (2004) found that providing opportunities for PSETs to reflect upon and discuss their own mathematical work is a valuable teaching tool in building their mathematical knowledge for teaching. The third-grade task that is the focus of this study is stated below:

I want to make a garden in the shape of a rectangle. I have 36 meters of fence for my garden. Using only whole number dimensions, what different-sized rectangular gardens can be made using all the fence (1m = 1cm). Record all the possible rectangular gardens on the recording sheet and determine which garden gives the least area and which one gives the largest area. Explain your thinking using words, symbols and diagrams.

PSETs were expected to solve this task using physical, visual and symbolic representations. After solving the task, they were expected to discuss how they made connections between the different types of representations used, what prerequisite knowledge a student should have prior to investigating the task and what mathematics they learned as a result of this exercise. They took turns to present their solutions to the rest of the class and their challenges, successes, misconceptions and errors were discussed. To maintain consistency in the teaching aspect of this study, the PSETs were expected to plan their lessons around the following learning goals:

1. Use different forms of representations to find the perimeter and area of rectangular figures;
2. Solve word problems involving area and perimeter of rectangles;
3. Find all possible areas of rectangle given a fixed perimeter using physical and visual representations.

3.3. Data Sources and Data Analysis

To answer the research question, various sources of data were collected: (a)videotapes of PSETs' teaching of the mathematics lessons; (b) PSETs' reflective journals; (c) field notes; and (d) solutions to the fixed perimeter garden problem. Overall, the analysis of the data followed the guidelines of Miles and Huberman (1997).

Analysis of PSETs' solution to the task was guided by the following questions by Ball et al. (2008): (a) Did they understand the task? (b) Did they use appropriate representations? (c) Were they able to generate all the possible rectangles using both concrete and pictorial representations (e.g., tiles and centimeter grid paper)? (d) Did they observe any patterns from the chart they completed? (e)Was the description of the patterns observed superficial or insightful? (e) Did they realize that the largest area is a square and that a square is a special type of rectangle?

As an illustration of PSETs' competence in using representations meaningfully, the following abilities by Huinker (2015) were considered: (1) being able to convey a mathematical idea in various forms; (2) knowing when and why it is appropriate to use particular mathematical representations; (3) being able to translate between and within modes of representations; (4) being able to us representations flexibly to solve problems. The PSETs' questioning strategies were analyzed using the four categories of questioning types recommended by NCTM (2014). They are information gathering, probing thinking, making mathematics visible and encouraging reflection and justification.

The reflective journals provided PSETs' the opportunity to reflect on their teaching after they taught the lessons and watched the recording of their teaching. The journals also served as a venue to synthesize aspects of the course they perceived as growth, successes and challenges. All the journal entries were collected every two weeks and read by the researcher. At the end of the study, the researcher reread the journal entries and identified all aspects of the course the PSETs identified as contributing to the development of SCK for teaching area and perimeter.

4.0. Findings

The elementary school setting, the mentor teachers, the elementary students, and the lecture sessions of the course provided the context for constructing PSETs' SCK for teaching area and perimeter in the domain of representational competence and questioning strategies. Although all these components directly or indirectly contributed to their knowledge construction, three themes emerged as the venues for discussing the research question. The three venues of learning were developing representational competence through the mathematical task, developing representational competence through teaching and developing questioning skills through teaching.

4.1. Representational Competence through Mathematical Task

Analysis of the PSETs' solutions to the task revealed that engaging with the mathematical task provided the PSETs the opportunity to develop a conceptual understanding of area and perimeter as they learned to use different representations to make sense of the problem. Although the three PSETs were aware that the 36 meters in the problem represented the perimeter, their initial attempt showed that they had a procedural knowledge of area and perimeter. They had learned to solve perimeter and area problems based on routine exercises without fully understanding the concept of area and perimeter. For example, Sandra explained in her initial attempt of the task that the dimensions of the rectangular garden were 12m x 3m, 9m x 4m, 36m x 1m, 18m x 2m and 6m x 6m and concluded that the rectangle with the largest area was 36m x 1m. "I basically applied the formula for finding the area of a rectangle. That is how I remember learning about area and perimeter. I couldn't use manipulatives to help me solve the problem" she explained.

Jessica indicated that after the initial unsuccessful attempts, "I used a tape measure and asked myself how I can break 36 inches into four parts so that the opposite sides have the same length to form a rectangle. This is when the light bulb came on. I used guess and check until I found three of the dimensions and then I noticed a pattern. Some of the patterns described by the participants included "the perimeter is constant which the area increases as you go down", "as the length decreases, the width increases", "the length goes down by one from 17 as the width goes up by one as you go down". After taking a closer look at her work, Jessica conjectured that "the difference between the length and the width is an even number (e.g., 17 – 1 = 16; 16 – 2 = 14; 15 – 3 = 12; and so on) and that the difference between the areas is always an odd number.

As you go up from the smallest area to the largest area, the difference between them is an odd number. From the areas 17, 32, 45, 56, 65, etc. you can see the difference between 17 and 32 is 15, between 32 and 45 is 13, between 45 and 56 is 11 and so on.

Maya noticed that as the length and width got closer the area got larger and larger and that the largest area is 80 square meters because the 9m by 9m is a square. This conclusion led to the discussion about a square being a special type of rectangle. They also thought that two rectangles with the same perimeter will have the same area. This misconception is consistent with research findings (Yeo, 2008).

Reflecting on her experience with respect to the task, Sandra wrote: "To teach a math topic well, you have to know the ins and outs of the problem and the math involved. I learned so much by doing the problem and seeing the different ways a problem can solved gives me more confidence. I have to make sure that I can do the math well before I teach it." Maya indicated that "creating each rectangle using the tiles, the grid paper and the chart was very important. Not only did I learn to use the physical objects and visuals, I was able to see how the tiles related to the drawings. Counting the number of tiles is the same as counting the number of squares in the rectangle."

4.2. Representational Competence through Teaching

Analysis of the videotapes and the researcher's field notes revealed that making the transition from how they learned mathematics particularly the concepts of area and perimeter did not occur smoothly. This challenge may be due in part to their procedural learning experiences. The PSETs demonstrated difficulties in using and linking the physical, visual and symbolic during the first two lessons. For example, Maya, after watching the video of her first lesson, reflected that although she learned to use the three forms of representations when she solved the task, she needed to build her confidence and practice using the visual and the physical representations in the subsequent lessons. This view, which was also shared by the other PSETs, is consistent with research findings that PSETs tend to experience some degree of mathematics anxiety even after been exposed to research-based effective teaching practices (Gresham, 2018). As Jessica pointed out in her reflective journal, "It is a challenge changing from the way I learned math which was using formulas to using different approaches like concrete materials and creating diagrams and pictures. I have to make conscious effort to relearn the math like what the course is teaching us."

During the third and fourth lessons, not only were the PSETs able use the different types of representations, they also facilitated discussions that enabled the students make connections among the different types of representations. For example, Sandra used Cuisenaire rods to help her students investigate and generate all the possible rectangles with a perimeter of 36 meters. After that the students used a centimeter grid paper to record and label the rectangles. Using probing questions, the students were able to deduce that the perimeter was L + L + W + W (i.e., symbolic representation). From the visual representation (i.e., centimeter grid paper) they were able to find the area. In addition, some of the students made the connection that the total number of squares was the same as multiplying two of the sides.

Maya used an inductive instructional process when she used colored tiles for students to explore simpler cases of the task. In particular, they examined the areas of rectangles with perimeters of 4, 6, 8, 10, …, 36 units. For each case, they represented the rectangle on a centimeter grid paper, labeled the dimensions and recorded their data in chart. These multiple modes of representations enabled students to make conclusions such as "the perimeter is always an even number"; "the perimeter cannot be less than 4 because a rectangle has 4 sides".

4.3. Developing Questioning Strategies Through Practice

In this section, I was interested in the way PSETs questioning strategies evolved over the course of the four lessons. This discussion is intended to highlight some of the critical events during the four lessons where the PSETs demonstrated growth in their questioning skills and where they appear to have faced some challenges. Although they included some questions that probed students' thinking, reflections and justifications in all their lesson plans, the video clips and observations showed that their first two lessons focused mostly on asking information gathering questions. These questions asked students to recall facts, procedures and definitions. For example, they asked questions such as: "What is the perimeter of the table?", "What answer do you have?" How did you figure out the perimeter?" "Can you write the formula for finding the area and the perimeter?" While these questions have a place in the process of constructing mathematical understanding, the goal of mathematics teaching and learning includes reasoning abstractly and quantitatively, constructing explanations and justifications, and making sense of mathematics (Huinker & Bill, 2017).

Analysis of the data showed that after viewing and discussing the videos of their first two lesson, PSETs got better at their questioning skills. For example, Maya began asking questions such as "How did you use the *Cuisenaire rods* to build the different rectangles?", What do you notice about the area and the sides of the rectangles?" Reflecting on her teaching, Maya indicated:

"As I watched my teaching, I realized that planning a lesson is one thing and teaching it is a different experience. Learning to teach math involves so many variables and I am glad that we focused these two ideas. I learned math traditional way and my teachers did most of the talking while we listened and did dittos. The discussion I have with the students depends on the type of questions I ask."

The video clips and field notes showed that by the time the PSETs got to the third and fourth lessons, about 50% of the questions they asked probed for students' thinking and encouraged reflections and justifications. For example, in

Jessica's fourth lesson she asked a student "which rectangle has the largest area?" the student responded by saying "the rectangle with the largest area is the square".
She immediately asked a follow-up question, "how do you know that the square has the largest area?" She wanted to find out if the student could support her answer with reasoning and justification.

4.4. Overall Experience

Making the transition from how they learned mathematics particularly the concepts of area and perimeter to thinking about area and perimeter conceptually did not occur smoothly. This challenge may be due in part to their procedural learning experiences. As Jessica pointed out: "It is a challenge changing from the way I learned math which was applying rules to using different approaches like concrete materials and creating diagrams and pictures. I have to make conscious effort to relearn the math like what the course has taught us."

PSETs' perceived the investigation of the *fixed perimeter garden task*, the teaching episodes and watching and discussing their teaching as key aspects of the course that contributed to the development of their SCK for teaching area and perimeter. The task encouraged PSETs to make and test conjectures and to look for relationships among the quantities and develop problem solving skills. The researcher felt that the task required students to thoughtfully review and think more deeply about the area-perimeter relationships as well as the properties of rectangle and square. For example, Jessica indicated:

"I liked the fact that we solved the problem because I didn't know that a square is a special type of rectangle. I would have told the students that the largest rectangle is the one with the area of 80 square meters and not 81 square meters since the one with area of 81 is a square."

"One of the lessons I learned from doing the problem is that as a teacher I have to solve the problem myself and use different strategies before I teach it to students" said Maya. Also, Sandra said: "I learned so much by solving the problem. It helped me experience the different ways a problem can be solved. This gave me a new perspective and helped build my confidence. I have to make sure that I understand the math well before I teach it."

Another aspect of the field-based mathematics methods course that all the PSETs found to have played a role in developing their SCK was the classroom experience particularly planning and teaching the four lessons. This experience "Gave me the opportunity to plan and teach a math lesson for the first time. I liked the fact that the lessons were video-taped because I was able to watch my own teaching and critique it" (Maya). Jessica indicated that planning and teaching the lesson sharpened her questioning skills and ways to introduce a lesson to students. This requires knowing the math and allowing students to think for themselves. I learned a lot about myself as a student and as a future teacher and what I need to work on."All three PSETs conveyed a sense of empowerment as a result of the classroom experience which was well put by Maya when she wrote: "This experience allowed me to apply what I learned in the course lectures to an actual teaching of students."

5.0. Discussions

This study documents the successes and challenges in developing PSETs' SCK for teaching area and perimeter. In particular, this study focused on the development of PSETs' representational competence and questioning strategies (Huinker, 2015; Ball et al., 2008). The finding suggests that the PSETs in this study entered the course with rule-bound knowledge of area and perimeter. They confused area and perimeter which is consistent with previous studies (e.g., Yeo, 2008; Menon, 1998). In spite of their procedural knowledge, the results appear to suggest that the PSETs exhibited growth in their conceptual understanding of area and perimeter. In addition, they developed skills in selecting and using different modes of representations to help construct their own mathematical knowledge and that of the students they taught.

One of the key sources of their growth in their SCK was the opportunity to first solve the task and then engage in a reflective analysis and discussion of their solutions. This is in concert with findings by Ryan and Williams (2007) who found that discussing PSETs' work samples and identifying their strengths, misconceptions and errors is an effective way to support the development of their MKT. Also, teaching the four lessons was significant to developing their representational competence. Although they lacked confidence in their ability in using physical, visual and contextual representations initially, they made a conscious effort to make some cognitive shift during the second half of their teaching experience. This shift may be attributed to their commitment to learning to teach differently from how they learned mathematics and the desire to be effective mathematics teachers. Since many PSETs enter teacher education programs with high levels of mathematics anxiety (Sloan, Daane, & Giesen, 2002), they may need multiple sources of experience to apply what they learned in the course lectures in an actual classroom setting.

Providing PSETs the opportunity to first solve the mathematics task allowed the researcher to identify and discuss any misconceptions and errors they held about area and perimeter prior to the planning and implementing lessons through the task. Their initial solutions to the task showed that they had procedural knowledge of area and perimeter rather than conceptual and relational understanding.

In addition, two of the three participants were confused about area and perimeter when she said that the 36 meters in the task represented the area. These findings are consistent with past research findings (e.g., Yeo, 2008; Ma, 1999; Menon, 1998). As the PSETs indicated in their reflective journals, engaging in a whole class discussion of their solutions helped them better understand the task which led to a deeper exploration of the task.

5.2. Conclusions

The purpose of this article is to investigate the impact of a field-based mathematics methods course on the development of PSETs' SCK for teaching area and perimeter in the domain of representational competence and questioning strategies. The findings suggest PSETs exhibited small but significant gains. This contributes to our understanding of how-to better support PSETs in the development of their mathematical knowledge for teaching. That is, by providing them with multiple experiences in engaging with mathematics tasks and planning and teaching lessons through a particular task. Also, providing them the opportunity to critique their teaching with the support of the course instructor contributed to their growth. While the current study focused on only area and perimeter, a methods course usually addresses many topics and thus has limited opportunities to delve deeper. A possible solution to this challenge is to select mathematical ideas that cut across various content areas of the mathematics curriculum (Ding, 2016).

Although the PSETs lacked confidence in asking high-order questions even when they had the opportunity, the findings appear to indicate that providing them with more planning and teaching opportunities hold promise in developing their SCK for teaching. As pointed out by Saunders, Nielson, Gall and Smith (1975), preservice teachers who engaged in regular microteaching produce more consistent and substantial gains in the use of questioning skills than pre-service teachers who received observation and lecture-discussion treatments. If the ultimate goal of teacher education is to improve the mathematics preparation of PSETs, providing them with both practical experience and opportunities to investigate tasks and plan and implement lessons through a specific mathematics idea might be necessary and helpful. One of the significant outcomes the PSETs gleaned from this field-based mathematics methods course was that learning to teach mathematics is a personal journey that requires among other characteristics self-determination, perseverance, commitment, flexible thinking and willingness to change. As such, they need encouragement, support and the resources to work toward continual, and incremental growth as they learn to teach mathematics.

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